#### Complex Prediction Problems A novel approach to multiple Structured Output Prediction

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Yasemin Altun Complex Prediction

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- Extract structured information from unstructured data
- Typical subtasks
  - Named Entity Recognition: person, location, organization names
  - Coreference Identification: noun phrases referring to the same object
  - Relation extraction: eg. Person works for Organization
- Ultimate tasks
  - Document Summarization
  - Question Answering

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- Complex tasks consisting of multiple structured subtasks
- Real world problems too complicated for solving at once
- Ubiquitous in many domains
  - Natural Language Processing
  - Computational Biology
  - Computational Vision

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- Motion Tracking in Computational Vision
- Subtask: Identify joint angles of human body



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#### **Complex Prediction Example**

- 3-D protein structure prediction in Computational Biology
- Subtask: Identify secondary structured Prediction from amino-acid sequence

AAYKSHGSGDYGDHDVGHPTPGDPWVEPDYGINVYHSDTYSGQW AAYKSHGSGDYGDHDVGHPTPGDPWVEPDYGINVYHSDTYSGQW



## Standard Approach to Complex Prediction

#### **Pipeline Approach**

- Define intermediate/sub-tasks
- Solve them individually or in a cascaded manner
- Use output of subtasks as *features* (input) for target task



where for POS and for NER where x:x+POS tags

#### • Problems:

- Error propagation
- No learning across tasks

#### New Approach to Complex Prediction

#### Proposed approach:

- Solve tasks jointly discriminatively
  - Decompose multiple structured tasks
  - Use methods from multitask learning
    - Good predictors are it smooth
    - Restrict the search space for smooth functions of all tasks
  - Device targeted approximation methods
    - Standard approximation algorithms do not capture specifics
    - Dependencies within tasks are stronger than dependencies across tasks

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- Advantages
  - Less/no error propagation
  - Enables learning across tasks

## Structured Output (SO) Prediction

Supervised Learning

• Given input/output pairs  $(x, y) \in \mathcal{X} \times \mathcal{Y}$ 

$$\mathcal{Y} = \{0, \ldots, m\}, \mathcal{Y} = \Re$$

- Data from unknown/fixed distribution D over  $\mathcal{X} \times \mathcal{Y}$
- Goal: Learn a mapping  $h: \mathcal{X} \to \mathcal{Y}$
- State-of-the art are discriminative, eg. SVMs, Boosting
- In Structured Output prediction,
  - Multivariate response variable with structural dependency.  $|\mathcal{Y}|$ : exponential in number of variables
  - Sequences, tree, hierarchical classification, ranking

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#### SO Prediction

- Generative framework: Model P(x, y)
  - Advantages: Efficient learning and inference algorithms
  - Disadvantages: Harder problem, Questionable independence assumption, Limited representation
- Local approaches: eg. [Roth, 2001]
  - Advantages: Efficient algorithms
  - Disadvantages: Ignore/problematic long range dependencies
- Discriminative learning
  - Advantages: Richer representation via kernels, capture dependencies
  - Disadvantages: Expensive computation (SO prediction involves iteratively computing marginals or best labeling during training)

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## Formal Setting

- Given  $S = ((x_1, y_1), \dots, (x_l, y_n))$
- Find  $h: \mathcal{X} \to \mathcal{Y}, h(x) = \operatorname{argmax}_{y} F(x, y)$
- Linear discriminant function  $F : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$

$$F_{w}(x,y) = \langle \psi(x,y), w \rangle$$

- Cost function:  $\Delta(y, y') \ge 0$  eg. 0-1 loss, Hamming loss
- Canonical example: Label sequence learning, where both x and y are sequences



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# Maximum Margin Learning [Altun et al 03]

Define separation margin [Crammer & Singer 01]

$$\gamma_i = F_w(x_i, y_i) - \max_{y \neq y_i} F_w(x_i, y)$$

• Maximize  $\min_i \gamma_i$  with small ||w||



• Minimize  $\sum_{i} \max_{y \neq y_i} (1 + F_w(x_i, y) - F_w(x_i, y_i))_+ + \lambda \|w\|_2^2$ 

$$\sum_{i} \max_{y \neq y_{i}} (1 + F_{w}(x_{i}, y) - F_{w}(x_{i}, y_{i}))_{+} + \lambda \|w\|_{2}^{2}$$

• A convex **non**-quadratic program

$$\min_{w,\xi} \frac{1}{2} \|w\|_2^2 + \frac{C}{n} \sum_i \xi_i$$

s.t.  $\langle \boldsymbol{w}, \psi(\boldsymbol{x}_i, \boldsymbol{y}_i) \rangle - \max_{\boldsymbol{y} \neq \boldsymbol{y}} \langle \boldsymbol{w}, \psi(\boldsymbol{x}_i, \boldsymbol{y}) \rangle \geq 1 - \xi_i, \quad \forall i$ 

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$$\sum_{i} \max_{y \neq y_{i}} (1 + F_{w}(x_{i}, y) - F_{w}(x_{i}, y_{i}))_{+} + \lambda \|w\|_{2}^{2}$$

A convex quadratic program

$$\min_{w,\xi} \frac{1}{2} \|w\|_2^2 + \frac{C}{n} \sum_i \xi_i$$

s.t. 
$$\langle \boldsymbol{w}, \psi(\boldsymbol{x}_i, \boldsymbol{y}_i) \rangle - \langle \boldsymbol{w}, \psi(\boldsymbol{x}_i, \boldsymbol{y}) \rangle \geq 1 - \xi_i, \quad \forall i, \forall \boldsymbol{y} \neq \boldsymbol{y}_i$$

- Number of constraints exponential
- Sparsity: Only a few of the constraints will be active

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• Using Lagrangian techniques, the dual:

$$\max -\frac{1}{2} \sum_{i,j,y,y'} \alpha_i(y) \alpha_j(y') \delta \psi(x_i, y) \delta \psi(x_j, y') + \sum_{i,y} \alpha_i(y)$$
  
s.t.  $0 \le \alpha_i(y), \quad \sum_{y \ne y_i} \alpha_i(y) \le \frac{C}{n}, \quad \forall i$ 

where  $\delta \psi(\mathbf{x}_i, \mathbf{y}) = \psi(\mathbf{x}_i, \mathbf{y}_i) - \psi(\mathbf{x}_i, \mathbf{y})$ 

- Use the structure of equality constraints
- Replace the inner product with a kernel for implicit non-linear mapping

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- Exploit sparseness and the structure of constraints by incrementally adding constraints (cutting plane algorithm)
- Maintain a working set  $\mathcal{Y}_i \subseteq \mathcal{Y}$  for each training instance
- Iterate over training instance
- Incrementally augment (or shrink) working set  $\mathcal{Y}_i$

$$\hat{y} = \underset{y \in \mathcal{Y} - y_i}{\operatorname{argmax}} F(x_i, y)$$
 via Dynamic Programming  
 $F(x_i, y_i) - F(x_i, \hat{y}) \le 1 - \epsilon$ ?

• Optimize over Lagrange multipliers  $\alpha_i$  of  $\mathcal{Y}_i$ 

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## Max-Margin Cost Sensitivity

- Cost function  $\Delta : \mathcal{Y} \times \mathcal{Y} \to \Re$ 
  - Multiclass 0/1 loss
  - Sequences Hamming loss
  - Parsing (1-F1)
- Extend max-margin framework for cost sensitivity
  - (Taskar et.al. 2004)

$$\max_{y\neq y_i}(\Delta(y_i,y)+F_w(x_i,y)-F_w(x_i,y_i))_+$$

• (Tsochantaridis et.al. 2004)

$$\max_{y\neq y_i}\Delta(y_i,y)(1+F_w(x_i,y)-F_w(x_i,y_i))_+$$

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#### **Example: Sequences**

- Viterbi decoding for argmax operation
- Decompose features into time  $\psi(x, y) = \sum_{t} (\psi(x_t, y_t) + \psi(y_t, y_{t-1}))$
- Two types of features
  - Observation-label:  $\psi(x_t, y_t) = \phi(x_t) \otimes \Lambda(y_t)$
  - Label-label:  $\psi(y_t, y_{t-1}) = \Lambda(y_t) \otimes \Lambda(y_{t-1})$

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Inner product between two features separately

$$\begin{split} &\langle \psi(\mathbf{x}, \mathbf{y}), \psi(\bar{\mathbf{x}}, \bar{\mathbf{y}}) \rangle \rangle \\ &= \sum_{s,t} \langle \phi(\mathbf{x}_t), \phi(\bar{\mathbf{x}}_s) \rangle \delta(\mathbf{y}_t, \bar{\mathbf{y}}_s) + \delta(\mathbf{y}_t, \bar{\mathbf{y}}_s) \delta(\mathbf{y}_{t-1}, \bar{\mathbf{y}}_{s-1}) \\ &= \sum_{s,t} k((\mathbf{x}_t, \mathbf{y}_t), (\bar{\mathbf{x}}_s, \bar{\mathbf{y}}_s)) + \tilde{k}((\mathbf{y}_t, \mathbf{y}_{t-1}), (\bar{\mathbf{y}}_s, \bar{\mathbf{y}}_{s-1})) \end{split}$$

- Arbitrary kernels on x
- Linear kernels on y

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## **Other SO Prediction Methods**

Find w to minimize expected loss E<sub>(x,y)~D</sub>[Δ(y, h<sub>f</sub>(x))]

$$w^* = \operatorname*{argmin}_{w} \sum_{i=1}^{l} \mathcal{L}(x_i, y_i, w) + \lambda \|w\|^2$$

- Loss functions
  - Hinge loss

• Log-loss: CRF [Lafferty et al 2001]

$$\mathcal{L}(x, y, f) = -F(x, y) + \log \sum_{\hat{y} \in \mathcal{Y}} \exp(F(x, \hat{y}))$$

Exp-loss: Structured Boosting [Altun et al 2002]

$$\mathcal{L}(x, y, f) = \sum_{\hat{y} \in \mathcal{Y}} \exp(F(x, \hat{y}) - F(x, y))$$

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## **Complex Prediction via SO Prediction**



• **Possible Solution:** Treat complex prediction as a loopy graph and use standard approximation methods

#### Shortcomings:

- No knowledge of graph structure
- No knowledge that tasks defined over same input space

#### Solution:

- Dependencies within tasks more important than dependencies across tasks. Use this for approximation method
- Restrict function class for each task via learning across tasks

# Joint Learning of Multiple SO prediction [Altun 2008]

- Tasks 1,..., *m*
- Learn a discriminative function  $T : \mathcal{X} \to \mathcal{Y}^1 \times \ldots \mathcal{Y}^m$

$$T(x,y;w,\bar{w}) = \sum_{\ell} \left[ F^{\ell}(x,y^{\ell};w_{\ell}) + \sum_{\ell'} F^{\ell\ell'}(y^{\ell},y^{\ell'};w,\bar{w}) \right]$$

where  $w_{\ell}$  capture dependencies within individual tasks  $\bar{w}_{\ell,\ell'}$  capture dependencies across tasks

- $F^{\ell}$  defined as before
- *F<sup>ℓℓ'</sup>* linear functions wrt cliques assignments of tasks ℓ, ℓ'

## Joint Learning of Multiple SO prediction

 Assume a low dimensional representation ⊖ shared across all tasks [Argyriou et al 2007]

$$F^{\ell}(\boldsymbol{x}, \boldsymbol{y}^{\ell}; \boldsymbol{w}_{\ell}, \Theta) = \left\langle \boldsymbol{w}_{\ell\sigma}, \Theta^{T} \psi(\boldsymbol{x}, \boldsymbol{y}^{\ell}) \right\rangle$$

• Find T by discovering  $\Theta$  and learning  $w, \bar{w}$ 

$$\min_{\Theta, w, \bar{w}} \hat{r}(\Theta) + r(w) + \bar{r}(\bar{w}) + \sum_{\ell=1}^{m} \sum_{i=1}^{n} \mathcal{L}^{\ell}(x_i, y_i^{\ell}; w, \bar{w}, \Theta),$$

- r, r regularization, eg. L2 norm
- $\hat{r}$ , eg. Frobenius norm, trace norm
- *L* loss function, eg. Log-loss, hinge-loss
- Optimization is **not** jointly convex over  $\Theta$  and  $w, \bar{w}$

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Via a reformulation, we get a jointly convex optimization

$$\min_{A,D,\bar{w}}\sum_{\ell\sigma} \left\langle A_{\ell\sigma}, D^+ A_{\ell\sigma} \right\rangle + \bar{r}(\bar{w}) + \sum_{\ell=1}^m \sum_{i=1}^n \mathcal{L}^{\ell}(x_i, y_i^{\ell}; A, \bar{w}).$$

- Optimize iteratively wrt  $A, \bar{w}$  and D
- Closed form solution for D

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- A and  $\bar{w}$  decomposes into tasks parameters
- Optimize wrt each tasks parameters iteratively
- **Problem**:  $F^{\ell,\ell'}$  is function of all other tasks
- **Solution**: Loopy Belief Propagation like algorithm where each clique assignment is approximated wrt current parameters iteratively
- Run DP for all other tasks, fix clique assignment values, optimize wrt current task

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#### Algorithm 1 Joint Learning of Multiple Structure Prediction Tasks

#### 1: repeat

- 2: for each task  $\ell$  do
- 3: compute  $\hat{a}_{\ell} = \operatorname{argmin}_{a} \sum_{i} \mathcal{L}(\mathbf{x}_{i}, \mathbf{y}^{\ell}; a) + \langle a, D^{+}a \rangle$  via computing  $\psi$  functions for each  $\mathbf{x}_{i}$  with dynamic programming
- 4: end for

5: compute 
$$D = \frac{(AA^T)^{\frac{1}{2}}}{\|A\|_T}$$
 and  $D^+$ 

6: until convergence

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- Task1: POS tagging evaluated with accuracy
- Task2: NER evaluated with F1 score
- Data: 2000 sentences from CONLL03 English corpus
- Structure: Sequence
- Loss: Log-loss (CRF)

|     | Cascaded   |              |               | Joint (no $\Theta$ ) | MT-Joint |
|-----|------------|--------------|---------------|----------------------|----------|
| POS | 92.63      |              |               | 93.21                | 93.67    |
| NER | 58.77(noP) | 67.42(predP) | 69.75 (trueP) | 68.51                | 70.01    |

Table 1: Comparison of cascaded model and joint optimization for POS tagging and NER

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- IE involves complex tasks, ie. multiple structured prediction tasks
- Structured prediction methods include CRFs, Max-Margin SO
- Proposed a novel approach to joint prediction of multiple SO problems
  - Using a special approximation algorithm
  - Using multi-task methods
- More experimental evaluation is required

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